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2006 J. Phys. A: Math. Gen. 39 11065

(http://iopscience.iop.org/0305-4470/39/35/008)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 11065–11074 doi:10.1088/0305-4470/39/35/008

Influence of the gravitational field on the quantum-nondemolition measurement of atomic momentum in the dispersive Jaynes–Cummings model

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Received 30 May 2006, in final form 24 July 2006
Published 11 August 2006
Online at stacks.iop.org/JPhysA/39/11065

Abstract

We present a theoretical scheme based on an su(2) dynamical algebraic structure to investigate the influence of a homogeneous gravitational field on the quantum-nondemolition measurement of atomic momentum in the dispersive Jaynes–Cummings model. In the dispersive Jaynes–Cummings model, when detuning is large and the atomic motion is in a propagating light wave, we consider a two-level atom interacting with the quantized cavity field in the presence of a homogeneous gravitational field. We derive an effective Hamiltonian describing the dispersive atom–field interaction in the presence of the gravitational field. We investigate the influence of the gravitational field on both the momentum filter and momentum distribution. Particularly, we find that the gravitational field decreases both the tooth spacing of momentum and the tooth width of momentum.

PACS numbers: 42.50.Ct, 42.50.Vk, 03.65.Ta, 03.75.Be

1. Introduction

Among the models describing the interaction between light and matter, the Jaynes–Cummings model (JCM) [1] seems to be ideal. The JCM describes the interaction between a two-level atom and a single quantized mode of the electromagnetic field in a lossless cavity within the rotating wave approximation (RWA). This is the simplest model of the radiation–matter interaction. It is simple enough to be exactly solved on the one hand and complicated enough to exhibit many fascinating quantum features on the other hand. These pure quantum effects include quantum collapses and revivals of atomic inversion [2], squeezing of the radiation field [3], atomic dipole squeezing [4], vacuum Rabi oscillation [5] and dynamical entangling and disentangling of the atom–field system in the course of time [6–8]. Further interest in the JCM comes from the fact that its theoretical predictions have been extensively used in the context of quantum information [9], atoms and ions trapping [10, 11] and quantum-nondemolition (QND) measurements [12].

In a general QND measurement [13], an observable signal of a quantum system is measured by detecting a change in an observable of the probe system coupled to the quantum system during the measurement time, without perturbing the subsequent evolution of the observable signal. We can therefore make a sequence of precise measurements of an observable signal such that the result of each measurement is completely predictable from the result of the preceding measurement. Such an observable is called a QND observable. Original QND ideas involved a dispersive coupling of the signal field to a material probe [14]. The QND method is quite generally based on dispersive and nonlinear effects. Methods to avoid the back action of the measurement on the detected observable have been proposed and implemented in the optical domain [15, 16]. These experiments are the realization of the QND schemes introduced in [14]. They rely on nonlinear coupling of the signal field to be measured with a probe field whose phase is altered by a quantity depending on the number of photons in the signal beam. In a paper by Sleator and Wilkens [17] a complementary scheme has been proposed in which a quadrature component of a propagating laser wave acts as the probe for the QND measurement of the atomic momentum. It is based on the Doppler effect on the component of atomic momentum along the propagation direction of the light field.

On the other hand, experimentally, atomic beams with very low velocities are generated in laser cooling and atomic interferometry [18]. It is obvious that for atoms moving with a velocity of a few centimetres or metres per second for a time period of several milliseconds or more, the influence of Earth's acceleration becomes important and cannot be neglected [19]. For this reason it is of interest to study the temporal evolution of a moving atom simultaneously exposed to the gravitational field and a single-mode travelling wave field. Since any quantum optical experiment in the laboratory is actually made in a non-inertial frame it is important to estimate the influence of Earth's acceleration on the outcome of the experiment. Recently, a semiclassical description of a two-level atom interacting with a running laser wave in a gravitational field has been studied [20].

In this paper we investigate a complementary scheme based on an su(2) dynamical algebraic structure to investigate the influence of the gravity on the QND measurement of atomic momentum in the dispersive JCM. In section 2, we present a full quantum treatment of the internal and external dynamics of the atom and find an alternative su(2) dynamical algebraic structure within the system. Based on this su(2) structure, we obtain an effective Hamiltonian describing the dispersive atom–field interaction in the presence of a gravitational field. Recently, the optical Schrödinger cat states have been realized in the dispersive JCM [21]. Also, these states have been verified experimentally, by Auffeves and coworkers, for a two-level atom interacting with a single mode of the electromagnetic field in the dispersive JCM [22]. In the dispersive JCM, the atom is in ground state and detuning is large, so one can neglect spontaneous emission. In section 3 we investigate the dynamical evolution of the system and show that how the gravitational field may affect the dynamical properties of the dispersive JCM. In section 4 we study the influence of gravitational field on the QND measurement of atomic momentum. Finally, we summarize our conclusions in section 5.

2. Dispersive Jaynes–Cummings model in the presence of gravitational field

In the dispersive JCM, we assume that the atom is in its ground state initially and we consider the case of large detuning so that the excited state of the atom is almost never populated, so we neglect spontaneous emission. The importance of dispersive JCM is because of its use in the generation of Schrödinger cat states with superposition of coherent states. These states have been prepared in various contexts. A great variety of methods have been proposed for generation of such states, for example, in a Mach–Zehnder interferometer [23] and one of the first schemes due to Yurke and Stoler [24] who showed that a coherent state propagating in a Kerr medium could lead to a Schrödinger cat state.

The system we consider here is a moving two-level atom exposed simultaneously to a single-mode travelling wave field and a homogeneous gravitational field. We take into account the atomic motion along the position vector $\hat{\vec{x}}$, so the evolution of the atom–field system in the presence of gravitational field and in the rotating wave approximation is governed by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} - M\vec{g}\cdot\hat{\vec{x}} + \hbar\omega_c\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{1}{2}\hbar\omega_{eg}\hat{\sigma}_z + \hbar\lambda[\exp(-\mathrm{i}\vec{q}\cdot\hat{\vec{x}})\hat{a}^\dagger\hat{\sigma}_- + \exp(\mathrm{i}\vec{q}\cdot\hat{\vec{x}})\hat{\sigma}_+\hat{a}], \tag{1}$$

where $\hat{a}$ and $\hat{a}^\dagger$ denote, respectively, the annihilation and creation operators of a single-mode travelling wave with frequency ω_c, $\vec{q}$ is the wave vector of the running wave and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels $|e\rangle$, $|g\rangle$ and Bohr transition frequency ω_{eg}. The atom–field coupling is given by the parameter λ and $\hat{\vec{p}}$, $\hat{\vec{x}}$ denote, respectively, the momentum and position operators of the atomic centre-of-mass motion and g is Earth's gravitational acceleration. We find that an alternative representation of su(2) algebra arises naturally from the system. In this manner, we construct a representation of su(2) algebra based on the generalized algebra and the Pauli matrices. From Hamiltonian (1), it is apparent that there exists an operator $\hat{K}$ which is constant of motion

$$\hat{K} = \hat{a}^\dagger\hat{a} + |e\rangle\langle e|. \tag{2}$$

In addition, the operator $\hat{a}\hat{\sigma}_+ = \hat{a}|e\rangle\langle g|$ commutes with $\hat{K}$. Now we introduce the following operators:

$$\hat{S}_0 = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|), \qquad \hat{S}_+ = \hat{a}|e\rangle\langle g|\frac{1}{\sqrt{\hat{K}}}, \qquad \hat{S}_- = \frac{1}{\sqrt{\hat{K}}}|g\rangle\langle e|\hat{a}^\dagger. \tag{3}$$

We can show that the operators $\hat{S}_0$, $\hat{S}_\pm$ satisfy the following commutation relations:

$$[\hat{S}_0, \hat{S}_\pm] = \pm\hat{S}_\pm, \qquad [\hat{S}_-, \hat{S}_+] = -2\hat{S}_0, \tag{4}$$

where $\hat{S}_0$, $\hat{S}_\pm$ are the generators of the su(2) algebra. In terms of su(2) generators, the Hamiltonian (1) can be rewritten as

$$\hat{H} = \frac{\hat{p}^2}{2M} - M\vec{g}\cdot\hat{\vec{x}} + \hbar\omega_c\hat{K} + \frac{1}{2}\hbar\triangle\hat{S}_0 + \hbar\lambda\sqrt{\hat{K}}(\exp(-\mathrm{i}\vec{q}\cdot\hat{\vec{x}})\hat{S}_- + \exp(\mathrm{i}\vec{q}\cdot\hat{\vec{x}})\hat{S}_+), \tag{5}$$

where

$$\triangle = \omega_{eg} - \omega_c, \tag{6}$$

is the usual detuning parameter.

Now we start to find the exact solution for the dynamical evolution of the total system governed by the Hamiltonian (5). The corresponding time evolution operator can be expressed as

$$\hat{u}(t) = \exp\left(\frac{\mathrm{i}M\vec{g}\cdot\hat{\vec{x}}t}{\hbar}\right)\hat{v}^\dagger\hat{u}_e(t)\hat{v}, \tag{7}$$

where

$$\hat{v} = \exp(-\mathrm{i}\vec{q}\cdot\hat{\vec{x}}\hat{S}_0), \tag{8}$$

$$\hat{u}_e = \exp\left(\frac{-\mathrm{i}\hat{H}_e t}{\hbar}\right). \tag{9}$$

It can be shown that the operator $\hat{u}_e(t)$ satisfies an effective Schrödinger equation governed by an effective Hamiltonian $\hat{H}_e$, that is

$$\mathrm{i}\hbar\frac{\partial\hat{u}_e}{\partial t} = \hat{H}_e\hat{u}_e, \tag{10}$$

where

$$\hat{H}_e = \frac{\hat{p}^2}{2M} - \hbar\hat{\triangle}(\hat{\vec{p}},\vec{g})\hat{S}_0 + \frac{1}{2}Mg^2t^2 + \vec{g}\cdot\hat{\vec{p}}t + \hbar\lambda(\sqrt{\hat{K}}\hat{S}_- + \sqrt{\hat{K}}\hat{S}_+) + \hat{H}_0, \tag{11}$$

with

$$\hat{H}_0 = \hbar\omega_c\hat{K} - \frac{\hbar}{2}\triangle\hat{S}_0 - \frac{q^2\hbar^2}{2M}\hat{S}_0 + \frac{q^2\hbar^2}{8M}, \tag{12}$$

and the operator

$$\hat{\triangle}(\hat{\vec{p}},\vec{g}) = \omega_c - \left(\omega_{eg} + \frac{\vec{q}\cdot\hat{\vec{p}}}{M} + \vec{q}\cdot\vec{g}t + \frac{\hbar q^2}{2M}\right), \tag{13}$$

has been introduced as the Doppler shift detuning at time t. Therefore, due to the Doppler shift of $\frac{\vec{q}\cdot\hat{\vec{p}}}{M}$, recoil frequency $\omega_{\mathrm{rec}} = \frac{\hbar q^2}{2M}$ and gravitational field, the detuning between the cavity field and the atomic transition frequency has been modified. The relevant time scale introduced by the gravitational influence is

$$\tau_a = \frac{1}{\sqrt{\vec{q}\cdot\vec{g}}}. \tag{14}$$

For an optical laser with $|\vec{q}| \simeq 10^7$ m^{-1} and Earth's acceleration $|\vec{g}| = 9.8$ m s^{-2}, τ_a is about 10^{-4} s. We remark that $\hat{\triangle}(\hat{\vec{p}},\vec{g})$ does only depend on the product $\vec{q}\cdot\vec{g}$, so that the influence of the gravitational acceleration on the internal evolution vanishes if the acceleration is perpendicular to the travelling field. Now we apply the interaction picture, i.e.,

$$\hat{u}_e = \exp\left(\frac{-\mathrm{i}t\hat{H}_0}{\hbar}\right)\hat{\hat{u}}, \tag{15}$$

such that

$$\mathrm{i}\hbar\frac{\partial\hat{\hat{u}}}{\partial t} = \hat{\hat{H}}\hat{\hat{u}}, \tag{16}$$

where

$$\hat{\hat{H}} = \frac{\hat{p}^2}{2M} - \hbar\hat{\triangle}(\hat{\vec{p}},\vec{g})\hat{S}_0 + \frac{1}{2}Mg^2t^2 + \hat{\vec{p}}\cdot\vec{g}t + \hbar(\hat{\kappa}(t)\sqrt{\hat{K}}\hat{S}_- + \hat{\kappa}^*(t)\sqrt{\hat{K}}\hat{S}_+), \tag{17}$$

and $\hat{\kappa}(t)$ is an effective coupling coefficient

$$\hat{\kappa}(t) = \lambda\exp\left(\frac{\mathrm{i}t}{2}\left(\hat{\triangle}(\hat{\vec{p}},\vec{g}) + \frac{\hbar q^2}{M}\right)\right). \tag{18}$$

In the limit of very small values of $|\langle\hat{\triangle}(\hat{\vec{p}},\vec{g})\rangle|$ and $\frac{\hbar q^2}{M}$, the coefficient $\hat{\kappa}(t)$ is independent of time. As it stands, the effective Hamiltonian (17) has the form of the Hamiltonian of the JCM, the only modification being the dependence of the detuning on the conjugate momentum and the gravitational field.

Now we consider the JCM in the dispersive limit and we obtain an effective Hamiltonian. In this limit, we assume that the atom is in its ground state initially and we consider the case of large detuning, $|\delta| \gg \kappa\sqrt{\langle\hat{a}^{\dagger}\hat{a}\rangle}$, with $\delta \equiv \omega_c - \omega_{eg} - \omega_{\rm rec}$. In this case, the excited state of the atom is almost never populated, so we neglect atomic spontaneous emission. In the interaction picture the transformed Hamiltonian (17) takes the following form:

$$\hat{\hat{H}}_{\rm int} = \exp\left(\frac{-{\rm i}\hat{\hat{H}}_0 t}{\hbar}\right)\hat{\hat{H}}_I \exp\left(\frac{{\rm i}\hat{\hat{H}}_0 t}{\hbar}\right), \tag{19}$$

where

$$\hat{\hat{H}}_0 = -\hbar\hat{\triangle}(\hat{\vec{p}}, \vec{g})\hat{S}_0, \tag{20}$$

$$\hat{\hat{H}}_I = \hbar(\hat{\kappa}\sqrt{\hat{K}}\hat{S}_- + \hat{\kappa}^*\sqrt{\hat{K}}\hat{S}_+) + \hat{H}(\hat{\vec{p}}, \vec{g}), \tag{21}$$

$$\hat{H}(\hat{\vec{p}}, \vec{g}) = \frac{\hat{p}^2}{2M} + \hat{\vec{p}}\cdot\vec{g}t + \frac{1}{2}Mg^2t^2. \tag{22}$$

Therefore we obtain

$$\hat{\hat{H}}_{\rm int} = \hbar(\hat{\kappa}\sqrt{\hat{K}}\hat{S}_- \exp(-{\rm i}t\hat{\triangle}(\hat{\vec{p}}, \vec{g})) + \hat{\kappa}^*\sqrt{\hat{K}}\hat{S}_+ \exp({\rm i}t\hat{\triangle}(\hat{\vec{p}}, \vec{g}))) + \hat{H}(\hat{\vec{p}}, \vec{g}). \tag{23}$$

Following Schleish [25], in the large detuning approximation, we arrive at the following effective Hamiltonian:

$$\hat{H}_{\rm eff} = \hat{H}(\hat{\vec{p}}, \vec{g}) + \hbar\hat{\Omega}(\hat{\vec{p}}, \vec{g})\hat{a}^{\dagger}\hat{a}, \tag{24}$$

where

$$\hat{\Omega}(\hat{\vec{p}}, \vec{g}) = \frac{|\hat{\kappa}|^2}{\hat{\triangle}(\hat{\vec{p}}, \vec{g})}, \tag{25}$$

is the momentum-dependent frequency of the harmonic oscillator and identified as the Doppler-modified ac stark shift of the atom–field interaction.

3. Dynamical evolution

In section 2, we obtained an effective Hamiltonian for the atom–field system in the presence of the gravitational field in the dispersive regime. In this section, we investigate the dynamical evolution of the system. We will show how the gravitational field may affect the dispersive JCM. We will also investigate the dispersive JCM, in the short- and long-time limits. The Schrödinger equation reads

$${\rm i}\hbar\frac{\partial|\psi\rangle}{\partial t} = \hat{H}_{\rm eff}|\psi\rangle, \tag{26}$$

where

$$|\psi(t)\rangle = |\psi_g(t)\rangle \otimes |g\rangle. \tag{27}$$

In the dispersive regime, we define $|\psi_g(t)\rangle$ as the state of the atomic centre-of-mass motion and the cavity field. We assume at $t = 0$, the atom–field system is described by the product state where the cavity field is initially prepared in a coherent state $|\alpha\rangle$. We apply evolution operator

$$\hat{u}(t) = \exp\left(\frac{-{\rm i}}{\hbar}\int_0^t \hat{H}_{\rm eff}(t')\,{\rm d}t'\right), \tag{28}$$

on the initial state

$$|\psi_g(t=0)\rangle = \left(\int {\rm d}^3p\,\phi_g(\vec{p})|\vec{p}\rangle\right) \otimes |\alpha\rangle, \tag{29}$$

where $\phi_g(\vec{p})$ is the probability amplitude for the centre-of-mass motion of the ground-state atom in the momentum representation, $\hat{\vec{p}}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle$. When the atom leaves the cavity after an interaction time τ, the state vector has evolved into the entangled state

$$|\psi_g(t=\tau)\rangle = \hat{u}(t=\tau)|\psi_g(t=0)\rangle$$
$$= \int \mathrm{d}^3 p \exp\left(\frac{-\mathrm{i}\tau p^2}{2M\hbar}\right) \exp\left(\frac{-\mathrm{i}\vec{p}\cdot\vec{g}\tau^2}{2\hbar}\right) \exp\left(\frac{-\mathrm{i}Mg^2\tau^3}{6\hbar}\right) \phi_g(\vec{p})|\vec{p}\rangle$$
$$\otimes |\alpha \exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle. \tag{30}$$

We now consider the gravitational influence on the dynamical evolution of the system for two limiting cases. The first, in the limit of small gravitational influence, $t \ll \tau_a$, means very small $\vec{q}\cdot\vec{g}$, i.e., the momentum transfer from the radiation field to the atom is only slightly altered by the gravitational acceleration because the latter is very small or nearly perpendicular to the travelling wave. In this limit, the state vector reads

$$|\psi_g(t=\tau)\rangle = \int \mathrm{d}^3 p \exp\left(\frac{-\mathrm{i}\tau p^2}{2M\hbar}\right) \phi_g(\vec{p})|\vec{p}\rangle \otimes |\alpha \exp(-\mathrm{i}\Omega(\vec{p})\tau)\rangle, \tag{31}$$

where

$$\Omega(\vec{p}) = \frac{|\kappa|^2}{\triangle(\vec{p})}, \qquad \triangle(\vec{p}) = \omega_c - \left(\omega_{eg} + \frac{\vec{q}\cdot\vec{p}}{M} + \frac{q^2\hbar}{2M}\right). \tag{32}$$

The Doppler shift detuning is independent of the gravitational field. The second, in the limit of long times, $t \gg \tau_a$, the atom is accelerated by Earth's gravity so that its velocity increases and the Doppler shift detuning in (13) depends on the gravitational field.

4. The QND measurement of atomic momentum

The QND measurement has been the subject of numerous investigations in the past two decades [26–34]. In a QND measurement the measurement-assisted perturbation of the system does not affect the observable that is desired to be determined, but is confined to other quantities. For such a measurement on a given system, the system must be coupled to another system (called probe), and an appropriately selected probe observable must be monitored during the measurement. The system–probe interaction has to be chosen in such a way that the corresponding interaction Hamilton commutes with the system observable. The interaction of radiation with a single atom involves both the electronic degrees of freedom and centre-of-mass degrees of freedom of atom. The interaction of a two-level atom with a standing laser field wave can result in a QND measurement of the atomic position. In [33] it has been shown that the position of an atom passing through a standing light wave is localized by making a quadrature phase measurement on the (sufficiently detuned) light field. This localization can be thought of as the creation of a virtual slit (or slits) for the atom by the field measurement. Moreover, it has been demonstrated [34] that one can measure the distribution of the transverse position of an atom crossing one or more optical cavities by monitoring the phase of the standing wave fields in the cavities. It has been shown that in the Kapitza–Dirac regime the method represents a QND measurement of the atomic position, and it can be applied to prepare narrow distributions of the transverse atomic position.

In the previous section we showed that how the gravitational field may affect the dispersive Jaynes–Cummings model. In this section we investigate the influence of gravitational field on the QND measurement of atomic momentum. The Hermitian quadrature phase operator $\hat{Y} = \frac{(\hat{a}+\hat{a}^\dagger)}{2}$ is used as a probe observable for a QND measurement of conjugate momentum $\hat{\vec{p}}$ since the Hamiltonian (17) satisfies the set of conditions: $\hat{\tilde{H}} = \hat{\tilde{H}}(\hat{\vec{p}})$, $[\hat{\tilde{H}}, \hat{\vec{p}}] = 0$, $[\hat{\tilde{H}}, \hat{Y}] \neq 0$. The probability for obtaining value Y for the quadrature phase $\hat{Y}$ may be expressed as

$$P(Y)\,\mathrm{d}Y = \mathrm{d}Y \int \mathrm{d}p|\phi_g(\vec{p})|^2 |\langle Y|\alpha \exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle|^2. \tag{33}$$

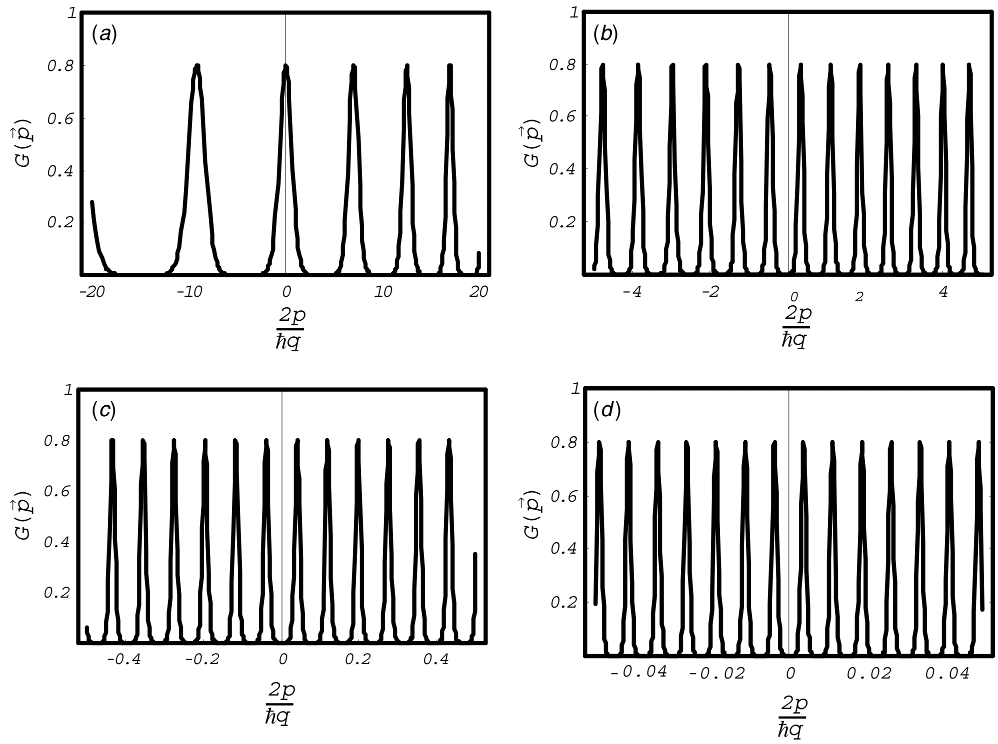

Figure 1. The momentum filter $G(\vec{p}) = |\langle Y|\alpha\exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle|^2$ as a function of $\frac{p}{\hbar q}$ that results a readout $Y = 0$. Here $(\frac{\kappa}{\delta})^2\tau\omega_{\mathrm{rec}} = 0.2$, $\tau\omega_{\mathrm{rec}} = 7.2$, $\kappa\tau = 140$, $\alpha = 2$, $q = 10^7\ \mathrm{m}^{-1}$, $M = 10^{-26}$ kg, $g = 9.8\ \mathrm{m\,s}^{-2}$. (*a*) $\tau = 14.4\times10^{-6}$ s, (*b*) $\tau = 14.4\times10^{-5}$ s, (*c*) $\tau = 14.4\times10^{-4}$ s and (*d*) $\tau = 14.4\times10^{-3}$ s.

The momentum distribution after a readout Y is given by the conditional probability $P(\vec{p}|Y)$ that the atom has a momentum vector $\vec{p}$:

$$P(\vec{p}|Y) = |\phi_g(\vec{p})|^2|\langle Y|\alpha\exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle|^2 P(Y)^{-1}, \tag{34}$$

where we assume

$$\phi_g(\vec{p}) = \frac{1}{\sqrt{2\pi\sigma_0}}\exp\left(\frac{-p^2}{\sigma_0^2}\right). \tag{35}$$

Straightforward calculation results in

$$\langle Y|\alpha\exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle = \left(\frac{2}{\pi}\right)^{\frac{1}{4}}\exp(-[|\alpha|\cos(\Omega(\vec{p}\cdot\vec{g})\tau-\varphi_\alpha)-Y]^2)$$
$$-2\mathrm{i}|\alpha|Y\sin(\Omega(\vec{p},\vec{g})\tau-\varphi_\alpha). \tag{36}$$

The momentum filter after a readout Y is defined as $G(\vec{p}) = |\langle Y|\alpha\exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle|^2$ with $\alpha = |\alpha|\exp(\mathrm{i}\varphi_\alpha)$.

In figures 1(*a*) and 2(*a*), respectively, we have plotted the momentum filter $|\langle Y|\alpha\exp(-\mathrm{i}\Omega(\vec{p},\vec{g})\tau)\rangle|^2$ and the momentum distribution $P(\vec{p}|Y = 0)$ as the functions of $\frac{p}{\hbar q}$ and for $Y = 0$. In these figures we have assumed $\left(\frac{\kappa}{\delta}\right)^2\tau\omega_{\mathrm{rec}} = 0.2$, $\tau\omega_{\mathrm{rec}} = 7.2$, $\kappa\tau = 140$, $\alpha = 2$, $q = 10^7\ \mathrm{m}^{-1}$, $M = 10^{-26}$ kg, $g = 9.8\ \mathrm{m\,s}^{-2}$, $\tau = 14.4\times10^{-6}$ s, $\sigma_0 = 1$ and $\varphi_\alpha = \Omega(0)\tau + \frac{\pi}{2}$ [11, 20]. Here we consider a beam of two-level atoms in the ground state with $\phi_g(\vec{p})$ given by (35) traversing in horizontal direction with the momentum

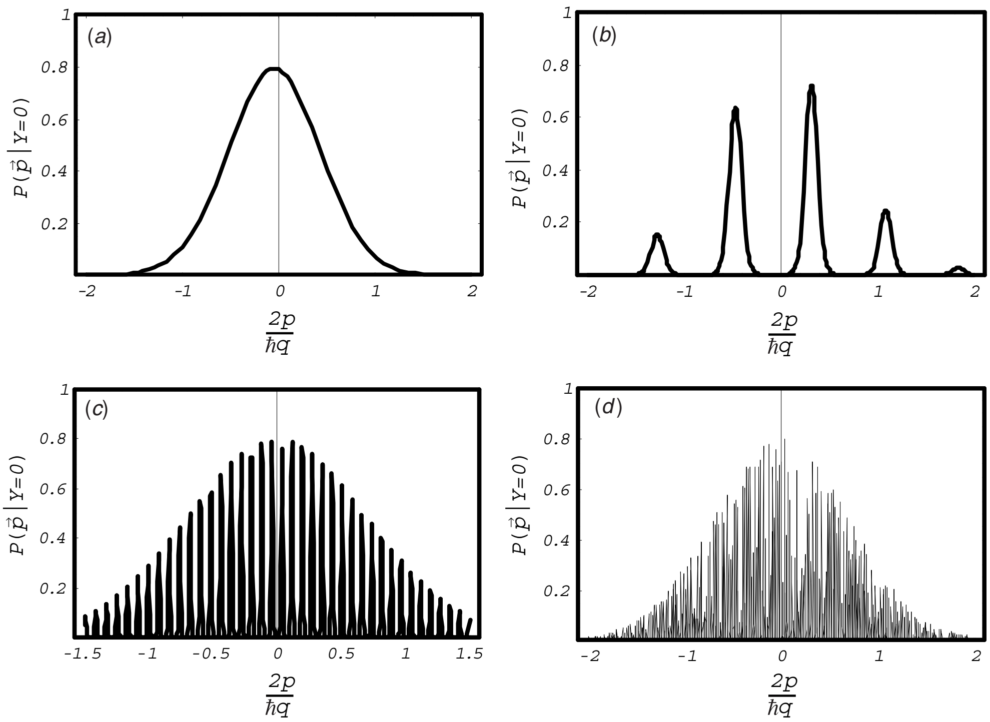

Figure 2. Momentum distribution after a readout $Y = 0$ has been detected. All parameters are the same as in figure 1. (*a*) $\tau = 14.4 \times 10^{-6}$ s, (*b*) $\tau = 14.4 \times 10^{-5}$ s, (*c*) $\tau = 14.4 \times 10^{-4}$ s and (*d*) $\tau = 14.4 \times 10^{-3}$ s.

vector $\vec{p}$ of an optical cavity in the presence of gravitational field so that $\vec{p} \cdot \vec{g} = 0$ and $\vec{p} \cdot \vec{q} = pq\cos\theta, \vec{q} \cdot \vec{g} = qg\sin\theta$, where θ is the angle between $\vec{q}$ and $\vec{p}$, and $\frac{\pi}{2} - \theta$ is the angle between $\vec{q}$ and $\vec{g}$. Before a given atom passes through the cavity, the cavity mode is prepared in the coherent state. After the atom passes through, its momentum is determined by a measurement of the cavity field. In figures 1(*b*)–(*d*) and 2(*b*)–(*d*), respectively, we plot the momentum filter and momentum distribution with respect to $\frac{p}{\hbar q}$ and for different times. These figures clearly show the influence of the gravitational field on the momentum filter and momentum distribution when the time increases. Furthermore, in figures 2(*a*)–(*d*) one can see oscillations. These oscillations result from quantum interference of translation motion with $\theta = \frac{\pi}{4}$ [25]. To estimate the spacing of momentum for small $\vec{p}$, we expand $\Omega(\vec{p}) \simeq \Omega(0) + \left(\frac{\kappa}{\delta - \vec{q}\cdot\vec{g}\tau}\right)^2 \frac{\vec{q}\cdot\vec{p}}{M}$ and obtain

$$\triangle\vec{p} = |\vec{p}_{n+1} - \vec{p}_n| = \hbar q \frac{\pi}{2}\left(\frac{\delta - \vec{q}\cdot\vec{g}\tau}{\kappa}\right)^2 \frac{1}{\omega_{\text{rec}}\tau}. \tag{37}$$

The slow variation of $\triangle\vec{p}$ in figures 1(*a*)–(*d*) is due to the nonlinearity of $\Omega(\vec{p}, \vec{g})$, which leads to a decreasing tooth spacing for increasing momenta. In a simple Gaussian approximation, the width of the teeth near $p = 0$ is given by

$$\sigma = \hbar q \frac{1}{4|\alpha|}\left(\frac{\delta - \vec{q}\cdot\vec{g}\tau}{\kappa}\right)^2 \frac{1}{\omega_{\text{rec}}\tau}. \tag{38}$$

From (37) and (38) one can see that the gravitational field decreases both $\triangle\vec{p}$ and σ.

5. Summary and conclusions

In this paper we have investigated the influence of the gravitational field on the dynamical behaviour of the dispersive JCM as well as on the QND measurement of atomic momentum. For this purpose, based on an su(2) algebraic structure, as the dynamical symmetry group of the model, we have derived an effective Hamiltonian describing the dispersive atom–field interaction in the presence of gravitational field. By finding an explicit form for the corresponding time evolution operator, we have explored the influence of gravity on the atom–field coupling and detuning parameter. We have shown that due to the gravitational field the atomic transition frequency experiences a Doppler shift and atom–field coupling becomes time dependent. Then we have investigated the influence of gravitational field on the QND measurement of atomic momentum in the dispersive JCM. We have shown that the gravitational field decreases both tooth spacing of momentum and the tooth width of momentum.

Acknowledgments

One of the authors (M M) wishes to thank The Office of Graduate Studies of the Science and Research Campus Islamic Azad University of Tehran for their support.

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